



MATLAB - Parallelization and Performance

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A word about Zoom

- Please stay muted unless necessary
- Questions: unmute, say "Question", remute, I will call you
 - -I will probably miss "Raise Hand" button
- Make sure microphone and screen sharing works
 - -Might be blocked by OS the first time
- Breakout rooms: groups of 2-3, do exercises collaboratively
 - -One person should share screen, rotate through





Round of introductions

- Who are you?
- What is your research about?
- What do you use MATLAB for?
- How much experience do you have with MATLAB?





Outline

- Part 1: Introduction
 - Background
 - -MATLAB at Uni Siegen
 - -MATLAB parallel features
- Part 2: MATLAB parallelization
 - -Using MATLAB on the cluster
 - Exercise 1
 - -Parallel pools and cluster profiles
 - Exercise 2
 - -Parallel programming in MATLAB
 - Exercise 3

- Part 3: Performance
 - Profiling basics
 - -Serial performance
 - -Parallel performance
 - Exercise 4
 - –Mex functions
- Summary

Lunch break (approx.)

- Around 12:30-13:00
- 60 minutes





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Background

- Lots of MATLAB users at Uni Siegen
 - -Sometimes very elaborate setups
- MATLAB not really built for HPC
 - -Rule of thumb: 1-2 orders of magnitude slower than C, Fortran
- However many features to support cluster use
- HPC community does not care much about MATLAB
- → What advice to give users?





Background: idea

- Idea: approach problem by "simulating" a regular user
- Create own MATLAB application
 - –More complex than code examples/exercises
 - -Simpler than real scientific codes
- Parallelize application
 - -Note pitfalls, tips & tricks
- Optimize application
 - Check against common HPC wisdoms
 - -Get a feeling for MATLAB performance





Test code, basics

- Code exists in multiple languages
 - -So far Python, MATLAB, Fortran
- 3D panel method (fluid dynamics)
 - -Based on APAME (open source), which is based on VSAERO (NASA)
 - -Simplified physics compared to modern CFD
- Mix of various parallel techniques
 - Understand techniques
 - Example for teaching purposes





Known source

strengths

Test code, equation system

• Equation system:

Unknown

dipole strengths

Known dipole influences

$$\underbrace{ \begin{pmatrix} C_{11} & C_{12} & C_{1N} \\ C_{12} & C_{22} & C_{2N} \\ C_{N1} & C_{N2} & C_{NN} \end{pmatrix}}_{Mat} \underbrace{ \begin{pmatrix} \mu_1 \\ \mu_2 \\ \mu_N \end{pmatrix}}_{x} = \underbrace{ - \begin{pmatrix} B_{11} & B_{12} & B_{1N} \\ B_{21} & B_{22} & B_{2N} \\ B_{N1} & B_{N2} & B_{NN} \end{pmatrix} \begin{pmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_N \end{pmatrix}}_{RHS}$$

• Operations:

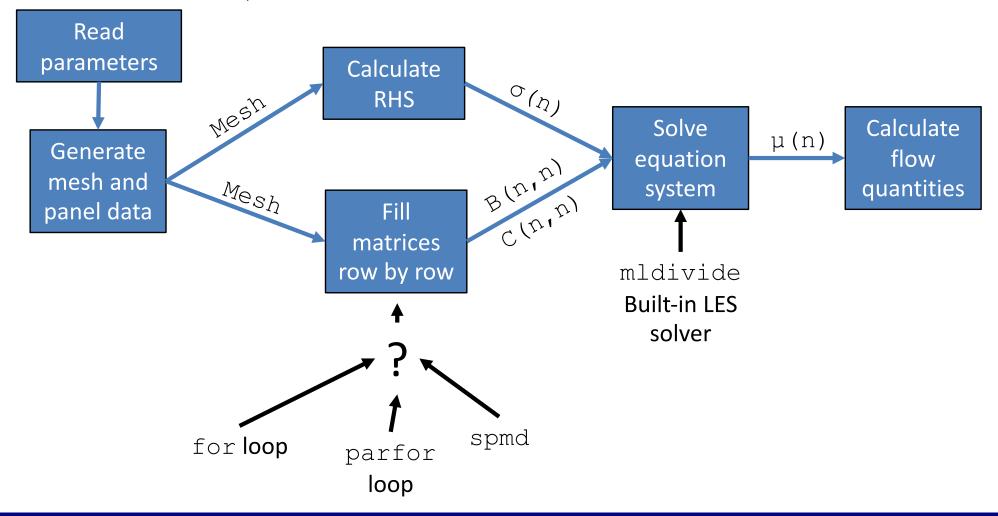
- -[Generate mesh]
- -Fill 2 matrices
- –Calculate RHS (matrix-vector product)
- -Solve linear equation system
- –[Calculate velocities]

Known source influences (incl. freestream)





Test code, data flowchart







Test code, operations

- Good mix of operations:
 - -Built-in mldivide for solving equation system
 - -Several built-in vecnorm function calls
 - -Some for loops
- Two compute-intensive phases:
 - -Fill B and C matrices
 - –Solve equation system
- Complexity roughly O(n^2)





Test code, data objects

- Data objects
 - -Mesh + Paneldata (mesh and panels)
 - -Influence data (infl)
 - Information needed to calculate influence of <u>every panel</u> onto <u>one point</u>
 - Lots of vectors (e.g. from every panel corner to target point)
 - Lots of scalars
 - -Singularity data (sing)
 - B and C matrix
 - Source and dipole strengths
 - RHS vector
 - -[Flow data] (velocities, pressure etc.)





Test code, data requirements

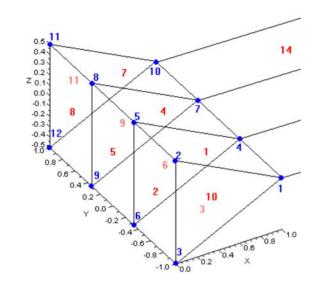
- Data object structure
 - –Constructor (allocates arrays)
 - -calc() method (fills arrays with values)
 - -Other methods as needed (called by calc() method)
- Total memory needed: (2 * N^2 + 100 * N) * 8 Byte
 - -More than 50 panels: matrices larger than everything else combined
 - -1000 panels \rightarrow 16 MB
 - -10000 panels \rightarrow 1.6 GB
 - -100000 panels → 160 GB





Test cases

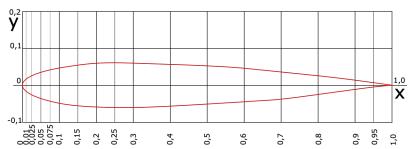
- 1. APAME test mesh
 - -Only 12 nodes, 11 panels
 - For debugging purposes
 - Reference solution exists
 - Wake ignored
 - -Triangles and quads



2. NACA 0012 airfoil

- –Simple geometry (one layer of quads)
- Can be mathematically generated
- -Easily scalable

NACA 0012







Investigation method

- Implement algorithm in Python
- Port to MATLAB (very easy)
- Set up MATLAB profiling
- Investigate serial + built-in multithreading performance
 - -What takes how much time?
 - —Is scaling logical?
 - Do common tips and tricks apply?
 - -What else did I notice?
- Parallelize with different strategies
 - -What is easy/hard to understand/implement
- Investigate parallel performance





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MATLAB licensing at Uni Siegen

- MATLAB available to everyone
 - But not free for everyone

- Three types of licenses
 - Employee license
 - -Student license
 - -Cluster license (Parallel Server)

No restriction on version





MATLAB licensing: employees

- Available to all institutes/workgroups, managed by Fak. 4
- Large collection of toolboxes (including Parallel Computing)
- May be installed on university or own computer
 - -May not be used on cluster outside Uni Siegen
- Cost (recently increased):

-1-3 users: €860 each

-4-6 users: €640 each

-7-10 users: €540 each

->10 users: 430 each





MATLAB licensing: cluster

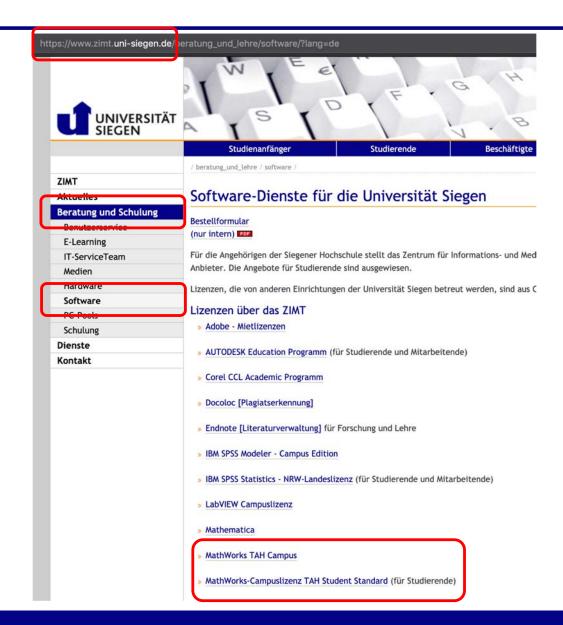
- MATLAB Parallel Server
- Formerly: MATLAB Distributed Computing Server
- 16 workers
- Not limited to one node
 - -With multithreading: use up to 16 nodes fully
- Parallel pool in SLURM job
 - -Pools in general are part of Parallel Computing Toolbox





Getting MATLAB

- ZIMT website
- Forms to fill out
- Detailed descriptions of license terms
- List of available toolboxes







MATLAB at Uni Siegen: miscellaneous info

- Free alternative: Octave also installed
 - Mostly MATLAB-compatible syntax
 - -Parallel package available
 - Have not looked at this in detail
- MATLAB training courses
 - Mech. Eng. lecture: "Introduction to Computer Science"
 - Contains MATLAB intro segment
 - -Many users at ENC
 - Other intro course?
- Getting help:
 - -MATLAB documentation free, extensive, also usage tips, tutorials
 - -Our cluster website explains integration, downloads





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Using MATLAB on the cluster: parallelism

- MATLAB on a cluster only makes sense if one uses parallelism
 - –More small computations
 - Larger computations
 - -Same computation in shorter time
- MATLAB parallel features:
 - -Built-in multithreading
 - -Parallel Toolbox
 - -MATLAB Parallel Server





MATLAB multithreading

- Built into MATLAB
 - -Since at least R2014
- Does not require user action
 - -However does allow some control
- You have used it without noticing
- Most built-in functions use it
 - -Example: mldivide()
- Allows use of a full node (or PC), but only one





MATLAB Parallel Toolbox

- Available as optional first-party toolbox
 - -Included in all Uni Siegen licenses
- Introduces concept of a "worker"
 - –Multiple MATLAB processes run in background ("workers"), one MATLAB process with GUI in foreground ("client")
- Concept of a "parallel pool"
 - Group of worker processes
 - -Work and data may be distributed among them
- Large arsenal of functions to distribute work over workers





MATLAB Parallel Server

- Server-(i.e. cluster-)side component
 - -User does not need additional license/tools etc. beyond Parallel Toolbox
- Allows starting a pool on multiple nodes
 - -Can talk to SLURM
- Separate license
 - -Uni Siegen has license for 16 workers
 - -Theoretically: run job on 16 nodes
- Introduces concept of "cluster profile"
 - "Local" profile identical to running workers on same PC
 - -ZIMT provides "HoRUS" profile





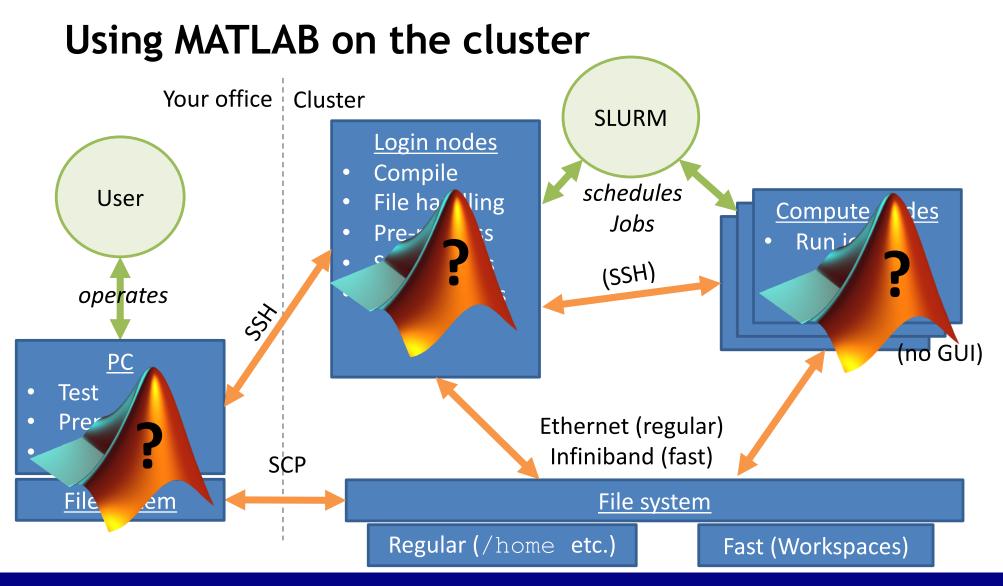
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Using MATLAB on the cluster

Different ways of using MATLAB

• Each has strengths and weaknesses

Each requires varying levels of setup

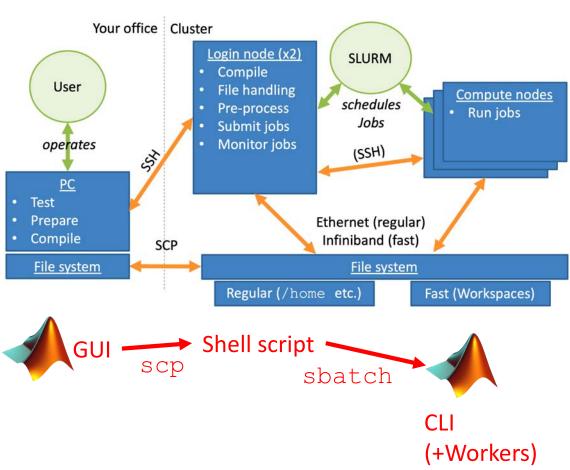
Five scenarios on following slides, with advantages and disadvantages





Using MATLAB: Scenario 1a

- You primarily use MATLAB on your PC
 - -Can use parallel pools locally
- Your computation is one M-file
- If you need more compute power, switch to cluster
 - –Copy M-file to cluster manually
 - -Create job script yourself
 - –Queue job like any other job







Scenario 1a: comparison

Advantages:

- Very simple job script
- Complete control over SLURM
- Easily allows other operations outside MATLAB
- Local parpool on node(s) still possible...
- Can use own PC for everything except extensive calculations

Disadvantages:

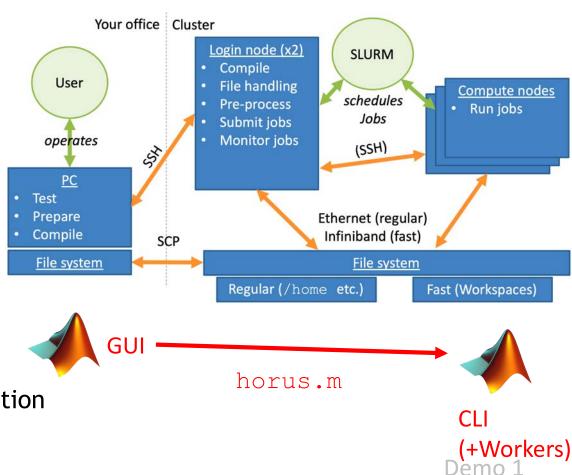
- No interactivity
- Manual file transfer necessary
- Manual job setup and queuing necessary
- ...but parpool needs second job (or only local)
- Heterogenous systems, toolboxes between cluster and PC





Using MATLAB: Scenario 1b

- Like scenario 1a, but partially automated
- Script donated by Prof. Nelles group, modified by me
- Automated steps:
 - Copying files to cluster
 - –Generating job script
 - Queuing job
 - -Retrieving files after completion
- Built-in documentation







Scenario 1b: comparison

Advantages:

- Highly automated
- Easy to use script
- Handles generation of parpool inside job if necessary
- Handles file synchronization between PC and cluster

Disadvantages:

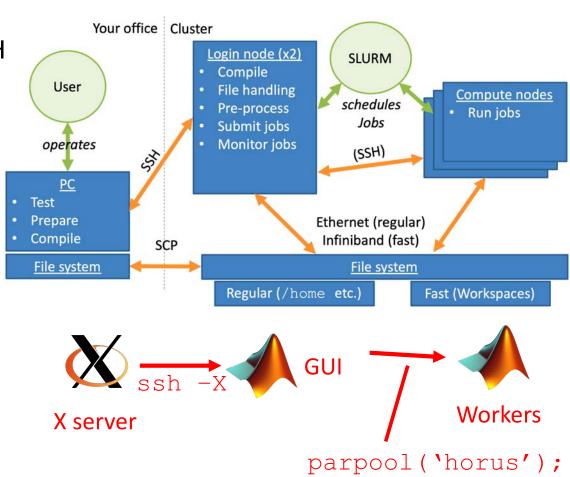
- Self-built solution (may contain bugs)
- Hard-coded job script, not flexible
- Still no interactivity inside job
- Still heterogeneity between PC and cluster
- Requires password-less login on Windows





Using MATLAB: Scenario 2a

- Connect to cluster via regular SSH
- Use MATLAB GUI on login node
 - –Edit scripts, brief test runs
 - -DO NOT RUN ACTUAL COMPUTATION ON FRONT-END
- Start pool from within MATLAB
 - –With horus profile
- Do compute-intensive work with pool (using Parallel Toolbox commands)







Scenario 2a: comparison

Advantages:

- Needs no MATLAB on PC
- Homogeneous between front-end and back-end
- Full interactivity (once job is running, you can do anything)
- No job script necessary: starting pool will automatically start job

Disadvantages:

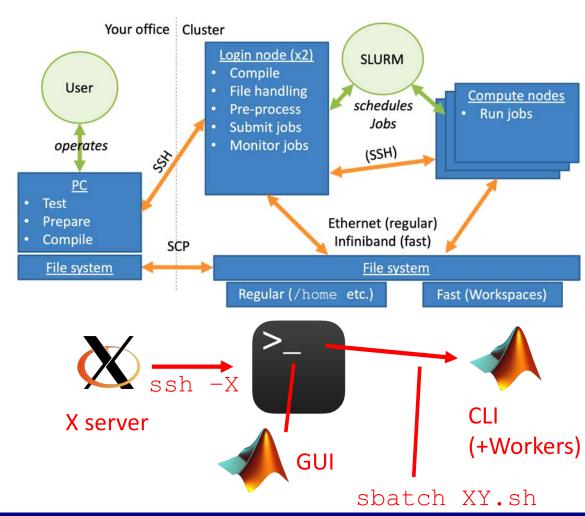
- Ugly GUI
 - -Does not even relaunch after close
- Have to wait for job to start
 - –Cannot run scripts
- Sometimes unstable SSH connection
- Occasional MPI problems in the past
- Control over SLURM (e.g. queue) limited/complicated





Using MATLAB: Scenario 2b

- Connect to cluster and open MATLAB GUI
 - -Like 2a
- Do not use MATLAB to launch jobs, only to edit scripts, prepare job
- Launch jobs separately via sbatch
 - -Like method 1a







Scenario 2b: comparison

Advantages:

- Combines advantages of 1a and 2a
 - –Flexibility
 - Homogeneity
 - Pool optional
- Can quickly look at job output files
- Uses local pool on nodes (no extra wait time)
- Multiple jobs at the same time

Disadvantages:

- Also combines disadvantages
 - -Ugly GUI
 - -Manual file transfer
 - –Manual job setup
 - Worker pool either only local or second job
- Little interactivity with job





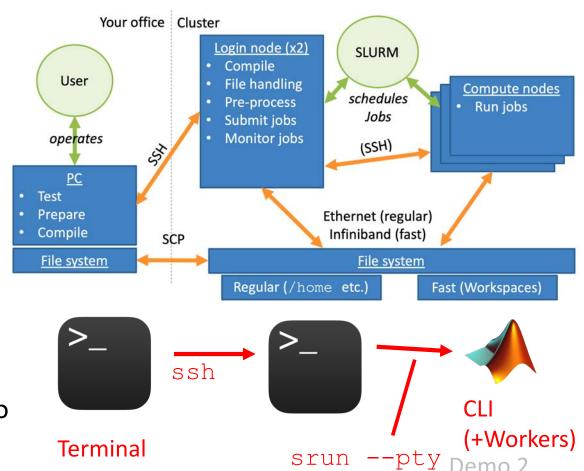
Using MATLAB: Scenario 3

You connect to cluster normally

Start an interactive job normally

Start CLI MATLAB inside job

• If needed, start pool inside of job







Scenario 3: comparison

Advantages:

- Compute resources reserved for you
- May still start pool with local profile
- Edit and run job on exact same MATLAB
- CLI MATLAB starts quickly, job continues on MATLAB crash

Disadvantages:

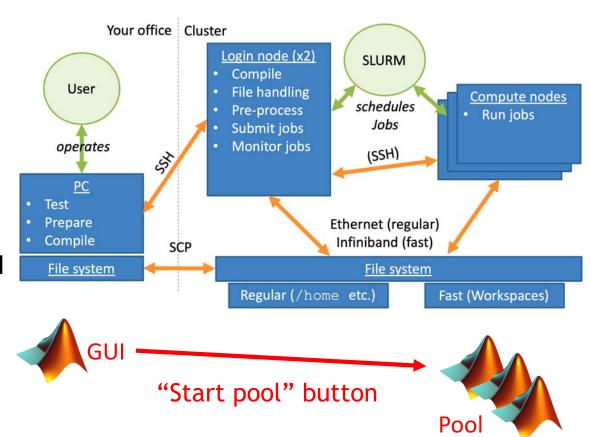
- Only CLI MATLAB available
- Initial job wait time
- Job dies if SSH connection to cluster closes





Not possible

- Cannot start pool on cluster from your PC
- Too much setup
- Only semi-officially supported
- We want you to think about resource usage







Simple basic job script

- Simple job script
 - -Scenario 1a, 2b
- Calls single MATLAB M script
- Script may open parallel pool internally
- If local:
 - -Can only run on same node
 - Needs different job settings

```
1 #!/bin/bash
2 #SBATCH -p short
3 #SBATCH -t 0:01:00
4 #SBATCH --nodes 1
5 #SBATCH --ntasks-per-node 1
6 #SBATCH --cpus-per-task 6
7 #SBATCH --mem 48000
8
9 matlab -nodisplay -r "demo1"
```





Simple basic job script

Job settings:

- One task (one MATLAB client process)
 - More tasks if workers started
- CPUs per task determines MATLAB threads
 - However MATLAB needs to be told this
- MATLAB called with
 - -nodisplay

```
1
    #!/bin/bash
    #SBATCH -p short
3
    #SBATCH -t 0:01:00
4
    #SBATCH --nodes 1
5
    #SBATCH --ntasks-per-node 1
    #SBATCH --cpus-per-task 6
6
7
    #SBATCH --mem 48000
8
9
    matlab -nodisplay -r "demo1"
```





Simple basic job script: multithreading

- MATLAB disregards --cpus-per-task by default and uses all cores on node
- Threads determined internally by maxNumCompThreads
- Can use SLURM environment variable inside MATLAB
 - -Either SLURM_CPUS_ON_NODE if only MATLAB running, or SLURM_CPUS_PER_TASK

Demo 3

```
disp('Default value of maxNumCompThreads:');
maxNumCompThreads

if ~isempty( getenv('SLURM_CPUS_ON_NODE') )
    maxNumCompThreads(str2num(getenv('SLURM_CPUS_ON_NODE')));
    disp('Taking value of maxNumCompThreads from SLURM:');
    maxNumCompThreads
end
```





Using a pool inside a job

- A job running MATLAB is still only one process
 - –Up to 12 cores via threads
- → Start a pool from inside job?
- Option A: use local profile
 - -Up to 12 workers
 - -MATLAB client also running: 11 workers?
- Option B: use 'horus' profile, accept that second job is started
 - -Up to 17 nodes theoretically (only if really needed please)
 - -Check resources: how much does MATLAB client need?





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Exercise 1

Login: schulungXY

(where XY=01-12)

PW: Matlab##Dec3

- Objectives
 - You can start MATLAB on the cluster
 - You understand the differences between the different approaches
- Tasks
 - Connect to cluster
 - -Start MATLAB in GUI mode and in CLI mode
 - -Start an interactive job with MATLAB
 - -Look at the output of maxNumCompThreads

Note the following slide (before you begin the exercise)





Exercise 1

- Notes
 - You have been given SLURM and Linux cheat sheets
 - You are allowed and indeed encouraged to use Google and the MATLAB documentation
 - -Interactive jobs explained on cluster website:
 https://cluster.uni-siegen.de/benutzung/jobs-einstellen/#interaktiv
- If bored, get creative
 - Play around with SLURM job settings
 - Open another Linux console with top, then run compute-intensive commands in MATLAB and watch CPU usage
 - -Look at horus.m

— ...





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MATLAB Multithreading

A word on maxNumCompThreads:

- Command to control number of threads
 - -Get: nThr = maxNumCompThreads
 - -Set: maxNumCompThreads (4)
 - Will return previous setting
- Cannot be set as MATLAB launch option that I am aware of
 - -Exception: -singleCompThread
- MATLAB documentation: "may be removed in future" (so far not)





Creating clusters and pools

Controlling parallel pool is done from inside MATLAB

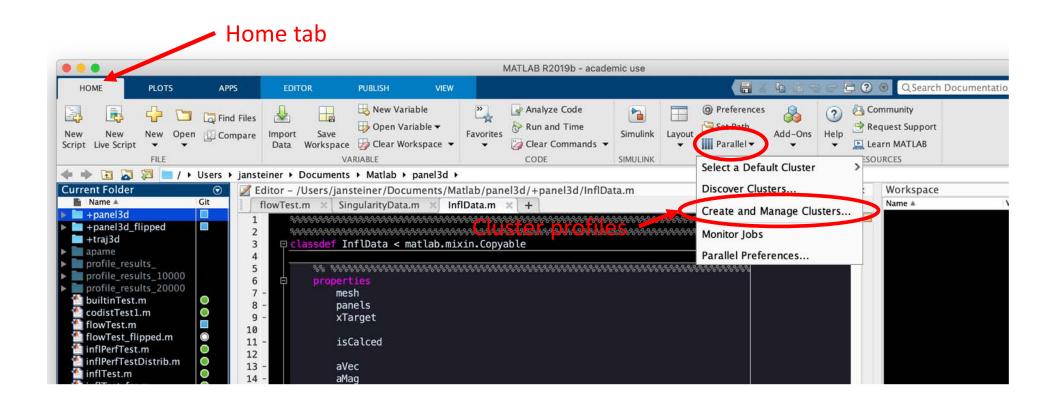
• Either in GUI or programmatically

- Main actions:
 - –Add and edit cluster profiles
 - -Start and stop parallel pool
 - -Send tasks to workers → next section





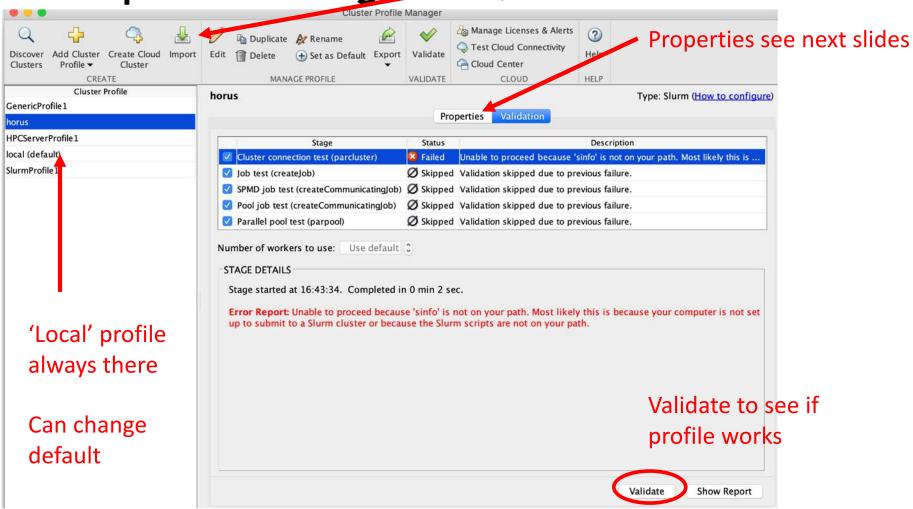
Cluster profile manager







Cluster profile manager Import button





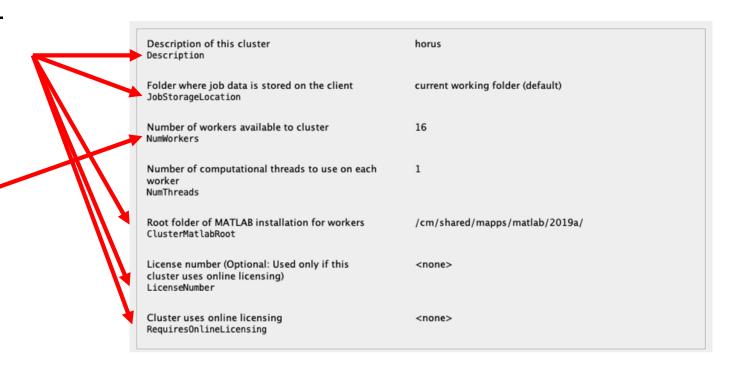


- We have prepared a cluster profile for HoRUS
 - Download from website
 - Import into MATLAB GUI or CLI on cluster
- You might have to adjust some options
 - -Some options might need adjusting on a case-by-case basis (e.g. walltime)
- Can be controlled programmatically (parcluster object)





- Pre-set and mostly selfexplanatory
 - -Leave these alone
- Maximum workers in a pool
 - -Can be at most 16
 - –May still start pool with fewer workers
 - -Can generally be left alone







- Maximum threads per worker
 - -Pre-set to 1
 - -Depends on your use case
 - Emphasis on built-in functions:12, but fewer workers
 - -Emphasis on pool: 1 thread + 16 workers
 - Full power needed: 16 workersx 12 threads
 - 16-node job → long wait time

	Description of this cluster Description	horus
	Folder where job data is stored on the client JobStorageLocation	current working folder (default)
\	Number of workers available to cluster NumWorkers	16
•	Number of computational threads to use on each worker NumThreads	1
	Root folder of MATLAB installation for workers ClusterMatlabRoot	/cm/shared/mapps/matlab/2019a/
	License number (Optional: Used only if this cluster uses online licensing) LicenseNumber	<none></none>
	Cluster uses online licensing RequiresOnlineLicensing	<none></none>





- Basic SLURM options
 - Pre-set to automatically start with correct number of tasks/cores
 - -Can be left alone
- Custom SLURM options
 - -Whatever you need
 - Especially queue and walltime
 - Unfortunately has to be changed in profile every time
 - -Common: --mem 48000

Resource arguments for job submission. Use the placeholders '^\^' for the number of workers and '^\^' for NumThreads. ResourceTemplate	ntasks=^N^cpus-per-task=^T^ (default)	
Additional command line arguments for job submission SubmitArguments	<none></none>	
Script that cluster runs to start workers CommunicatingJobWrapper	MathWorks provided script (default)	
FILES AND FOLDERS		
Automatically send code files to cluster. Data files must be listed below. AutoAttachFiles	true (default)	
Manually specify files and folders to copy from client to cluster nodes (One entry per line) AttachedFiles	<none></none>	
Manually specify folders to add to the workers' search path (One entry per line) AdditionalPaths	<none></none>	

ADDITIONAL SLURM PROPERTIES



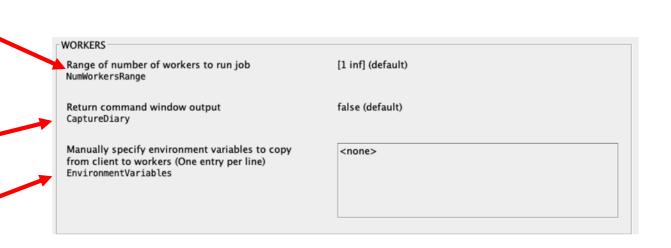


ADDITIONAL SLURM PROPERTIES Resource arguments for job submission. Use the --ntasks=^N^ --cpus-per-task=^T^ (default) placeholders 'ANA' for the number of workers and 'ATA' for NumThreads. ResourceTemplate Additional command line arguments for job <none> submission SubmitArguments Leave this alone Script that cluster runs to start workers MathWorks provided script (default) CommunicatingJobWrapper FILES AND FOLDERS Automatically send code files to cluster. Data true (default) files must be listed below. AutoAttachFiles Manually specify files and folders to copy from • Not sure what these do <none> client to cluster nodes (One entry per line) AttachedFiles -Can be left alone Manually specify folders to add to the workers' <none> search path (One entry per line) AdditionalPaths





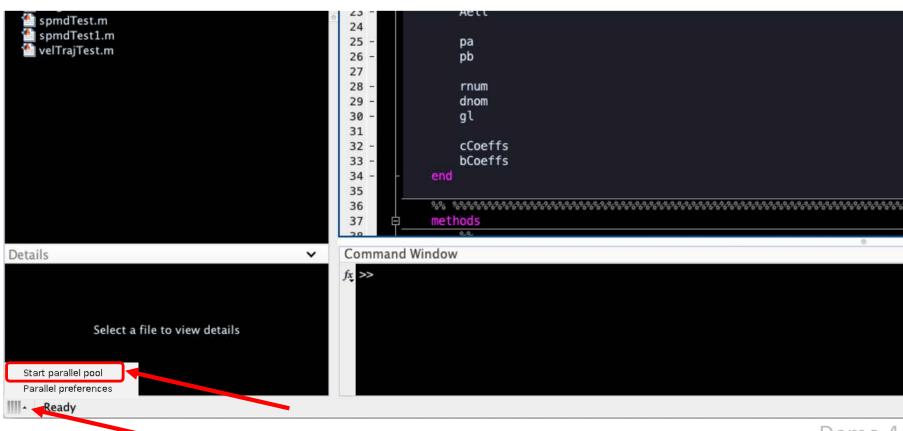
- Limit number of workers
 - -Normally not needed
- Not sure how this works exactly
- Might become important when using functionality external to MATLAB (MEX etc.)
 - Otherwise leave alone







Starting a pool from GUI



Demo 4





MATLAB commands for pool control

- Everything discussed so far can be set with MATLAB commands
- Object-oriented:
 - -Parallel pool object
 - Cluster object
- Important functions:
 - -p = parpool() starts a new pool
 - Optionally specify profile or workers or both:

```
p = parpool('horus',4)
```

- $-p = gcp() \rightarrow get current pool$
- -c = parcluster('profile') → get cluster object
 - Edit SLURM settings: c1.SubmitArguments = <string>





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Exercise 2

- Objectives
 - You understand the parallel pool settings
 - You have functioning local and Horus cluster profiles
- Tasks
 - -Open the cluster profile manager and examine the profile settings
 - -Import the horus_m2019 cluster profile (available at: https://cluster.uni-siegen.de/anwendungssoftware/matlab/#mdcs)
 - Launch a pool with the GUI button
 - -Launch pools with different settings programmatically

Note the following slide (before you begin the exercise)





Exercise 2

- Notes
 - -You should launch jobs in the short queue and with short walltimes to await waiting times
- If bored, get creative
 - -Examine parcluster and parpool object member variables
 - -Open another Linux console with top, then run compute-intensive commands in MATLAB and watch CPU usage
 - Examine the MATLAB "Parallel Preferences"

— ...





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Distributing tasks to workers

- Parallel functions (and associated concepts) in MATLAB
 - -Essentially mirrors usual parallel programming approaches
- Send task manually (not covered): batch(), parfeval()
- Parallel for loop: parfor
- Parallel program: spmd
- Interactive console on multiple workers: pmode
- Many MANY more
 - -E.g. Simulink: parsim/batchsim





The batch command

- batch command sends one task to one worker
- "Non-blocking": you can keep working on MATLAB client console
- Important concept: "future"
 - Worker completes work
 - -Stores results
 - -Future object is how to get results back
- Job "diary": log file etc.





Parallel for loops

- Very straightforward idea:
 - -Take a regular for loop
 - -Distribute loop iterations among workers
 - -Multiple iterations get done at the same time
- Simple to apply to existing code in principle...
 - –Will even start pool itself (not on HoRUS)
- ...however there are some complications
 - -Loop iterations have to be 100% independent, many implications
- MATLAB documentation very helpful here





Parallel for loops

• Basic parfor example

```
tic
n = 200;
A = 500;
a = zeros(1,n);
parfor i = 1:n
    a(i) = max(abs(eig(rand(A))));
end
toc
```

- Will run a few seconds
- Note tic and toc for time measurement

https://www.mathworks.com/help/parallel-computing/parfor.html





Parallel for loops: complications

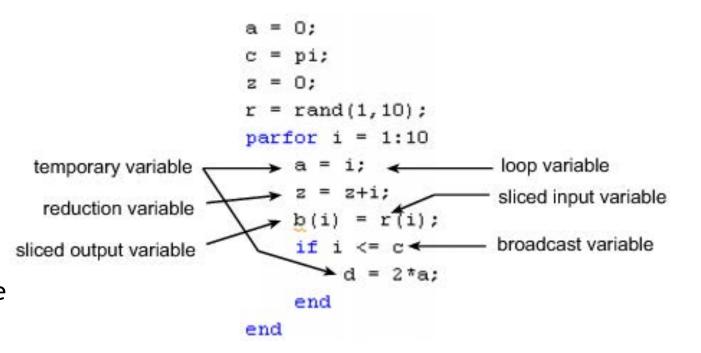
- Five different types of variables
- MATLAB has to tell which variable is which type
 - -Does it exist once or is it copied on each worker?
 - Does it change during the loop?
 - -What belongs to which worker?
- Error message if MATLAB cannot tell type
- Overall rule of thumb: the simpler the loop, the fewer problems
- Also useful for explaining parallel concepts in general





Parallel for loops: variable types

- 1. Loop variable
- 2. Sliced variable
- 3. Broadcast variable
- 4. Reduction variable
- 5. Temporary variable



https://www.mathworks.com/help/parallel-computing/troubleshoot-variables-in-parfor-loops.html





Parallel for loops: loop variables

1. Loop variable

- Loop index
- Must be consecutive increasing integer
- Cannot break out of loop early
 - Loop iterations executed in parallel
 - Order not clear
 - What is "early"?
- Cannot be modified

https://www.mathworks.com/help/parallel-computing/loop-variable.html





Parallel for loops: sliced variables

2. Sliced variable

- Exists only once
- Distributed over workers
 - Each worker gets different slice
- May be written to inside loop
- MATLAB has to know how to distribute

```
A = rand(1,10);
parfor ii = 1:10
    B(ii) = A(ii);
end
```

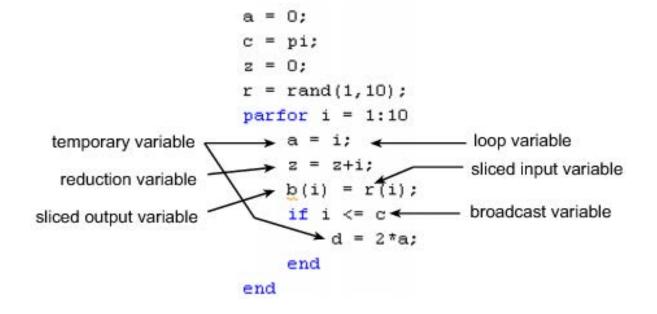
https://www.mathworks.com/help/parallel-computing/sliced-variable.html





Parallel for loops: broadcast variables

- 3. Broadcast variable
- Existing variable before parfor block
- Copied to each worker
- Cannot be modified inside parfor
- Caution with large variables



https://www.mathworks.com/help/parallel-computing/broadcast-variable.html





Parallel for loops: reduction variables

4. Reduction variable

- "Reduction": accumulate a value from each parallel task
 - Example: sum up value over each loop iteration
- Must be specific operation
- Cannot be dependent on iteration order

https://www.mathworks.com/help/parallel-computing/reduction-variable.html





Parallel for loops: temporary variables

5. Temporary variable

- Exists in each loop iteration separately
- Deleted at end of iteration
- If it exists before parfor, it is overwritten

https://www.mathworks.com/help/parallel-computing/temporary-variable.html





Code example: parallelizing a for loop

- LES matrix is filled row by row in loop
 - -Rows independent: good candidate for parallelization

- Influence data calculation (all panels on one point)
 - Self-contained operation

Result is copied into B and C matrices





Code example: parallelizing a for loop

```
% calcMatsFor()

for iTarget=1:nTarget

    xTarget = infl.panels.centers(iTarget,:);
    infl.calc( xTarget );

    cMat(:,iTarget) = infl.cCoeffs;
    bMat(:,iTarget) = infl.bCoeffs;

end

cMat = cMat.';
bMat = bMat.';
```

```
% calcMatsParfor()
parfor (iTarget=1:nTarget,nWorkers)
    xTarget = infl.panels.centers(iTarget,:);
    infl.calc( xTarget );

    cMat(:,iTarget) = infl.cCoeffs;
    bMat(:,iTarget) = infl.bCoeffs;
end

cMat = cMat.';
bMat = bMat.';
```

- Working parfor loop really possible with no changes
- Note how matrix is filled by column, then transposed





MATLAB pmode: introduction

- Parallel mode (pmode) is a parallel interactive console
 - -Start with pmode start
 - -Will open new window and start pool
- Similar to regular MATLAB console
 - -But sends command to each worker
- Also understands constructs like codistributed arrays
- Extremely practical for understanding parallel concepts
 - Also for debugging

Demo 6





MATLAB spmd construct: background

- SPMD: general parallel programming term
 - -<u>Single-Program Multiple-Data</u>
- Program is written once, run multiple times in parallel
- Trivial case: each instance does exactly the same thing
 - -More common: each does same task for different part of input data
 - -However anything is possible
- Often includes communication between instances





MATLAB spmd construct: introduction

- MATLAB: spmd block
 - Block contents run on workers
- More complicated, but also more flexible than parfor
- Variables either copied or distributed over workers
 - -Similar to parfor, but different mechanisms
- Important tools: labindex, numlabs
 - -Give ID of worker and total number of workers respectively
 - -Important to determine which worker works on what





MATLAB spmd: codistributed arrays

- Important concept: codistributed array
 - -Array (or n-D matrix), treated mostly like regular array by MATLAB
 - -But each part lives on different worker
 - -User can say which part lives where
 - Parallel programming term: Partitioned Global Address Space (PGAS)
- Distinction: distributed vs. <u>co</u>distributed array
 - Distributed arrays not covered
- Also common: composite objects
 - Object distributed over workers
 - Not covered here





MATLAB codistributed arrays

- SPMD construct requires moving data to workers manually
- Codistributor object: contains information on how array is distributed
 - -Can be along any axis (codistributor1d)
 - -Can also be block-cyclic (codistributor2dbc)
- Is otherwise treated like normal array
 - Makes things easier for user
 - -Sometimes tricky to keep track which array is codist or not
- SPMD performance: need to make local copy

Demo 7





MATLAB SPMD code example

- Info about array distribution is retrieved
 - –Which rows are on this worker?
- Local copies of part of array
 - -Codist-array could be addressed with (:,:)
 - –But: prohibitively expensive
- Local copies are written back to original array

```
codist1 = getCodistributor(cMat);
iGlob = codist1.globalIndices(1);
cLocal = getLocalPart( cMat );
bLocal = getLocalPart( bMat );
nLocal = size(cLocal,1);
for iTarget = 1:nLocal
     xTarget = infl.panels.centers(iGlob(iTarget),:);
    infl.calc( xTarget );
    cLocal(iTarget,:) = infl.cCoeffs;
    bLocal(iTarget,:) = infl.bCoeffs;
end
cMat(iGlob,:) = cLocal;
bMat(iGlob,:) = bLocal;
```





Second code example: solving the LES

 In addition to filling the LES, solving has to also be parallelized

 Even serial version needs modification (input data could be codistributed)

 Parallel solver needs to be inside spmd block

 Data needs to be scattered, then gathered again

```
% calcDoublets
doublets(:) = gather( mldivide(cCoeffs,rhs) );
% calcDoubletsParallel
spmd
    cCoeffsDist = codistributed(cCoeffs);
    rhsDist = codistributed(rhs);

solDist = mldivide(cCoeffsDist,rhsDist);
end
doublets(:) = gather( solDist(:) );
```





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- Part 3: Performance
 - Profiling basics
 - -Serial performance
 - -Parallel performance
 - Exercise 4
 - –Mex functions
- Summary





Exercise 3

- Objectives
 - You understand the parfor construct
 - -You understand the spmd construct and codistributed arrays
- Tasks
 - -Open pmode and create some codistributed arrays
 - -Implement the parfor example (Appendix A) and run it
 - -Implement the spmd example (Appendix B) and run it

Note the following slide





Exercise 3

- Notes:
 - -You can leave out the commented code in the examples for now
- If bored, get creative
 - Add new variables into parfor
 - Add new non-distributed, distributed and codistributed arrays to both examples

— ...





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Serial Performance, profiling

- Never optimize without measurement
- Simple stopwatch: tic and toc
 - -tic starts timer, toc ends it
 - –Can have multiple timers
- MATLAB profiler:
 - -Simply use profile on and profile off
 - -How often was each function called, how much time was spent in it
 - -Save HTML files with profsave
 - -Complication for cluster: MATLAB directory is hardcoded in HTML → image files not found

Demo 8





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Serial Performance

- How do I make MATLAB faster?
 - -Throw out MATLAB and use a real programming language!
 - -Barring that, use built-in functions
- Remember: never optimize without measurement!
- Common performance wisdoms in other languages:
 - -Minimize reallocation (e.g. C)
 - Avoid explicit loops (e.g. Python)
 - Use library functions (BLAS, LAPACK)
 - Be aware whether the language is row-major or column-major





Serial Performance, net vs. gross memory

- What is memory use of a Matlab array?
 - Number of entries times 8 byte (double precision float)
 - Plus overhead
 - –So what is overhead?

- Gross memory usage: Matlab will not tell you, but OS will
 - -Mac OS and Linux: 1.5 2 GB when idle
 - Rough approximation





Serial Performance, common wisdoms

- Test: how much is the difference
 - 1. Minimize allocations, reallocations, copies
 - 2. Avoid explicit loops
 - 3. Use built-in functions
- Profile built-in function vs. explicit loops
 - -With and without reallocation

- Result:
 - -Built-in functions two orders of magnitude faster
 - -Reallocation vs. no reallocation makes little difference

Demo 9





Serial Performance, common wisdoms

- Minimize allocations, reallocations, copies
 - -Difference less than expected
 - -That is not a good thing
- Avoid explicit loops
 - -Oh boy, yes!
- Use built-in functions
 - Yes (because it's not MATLAB)

Best performance when using obvious way (that is a good thing)





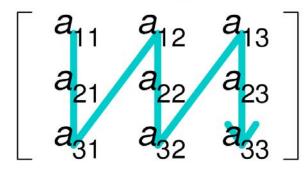
Serial Performance, common wisdoms

- Be aware whether language is row-major or column-major
 - —Official: Matlab is columnmajor (like Fortran)
 - -Test: flipped all indices
 - -Reality: "right way" is slower
 - Sometimes same time
 - Only exception: vecnorm() faster

Row-major order

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

Column-major order







Serial Performance, other stuff

- n threads never means n times the performance
 - -Overhead
 - -General problem in HPC

- Best friend for debugging: maxNumCompThreads (n)
 - -Sets number of threads to use
 - -Default same as CPUs available



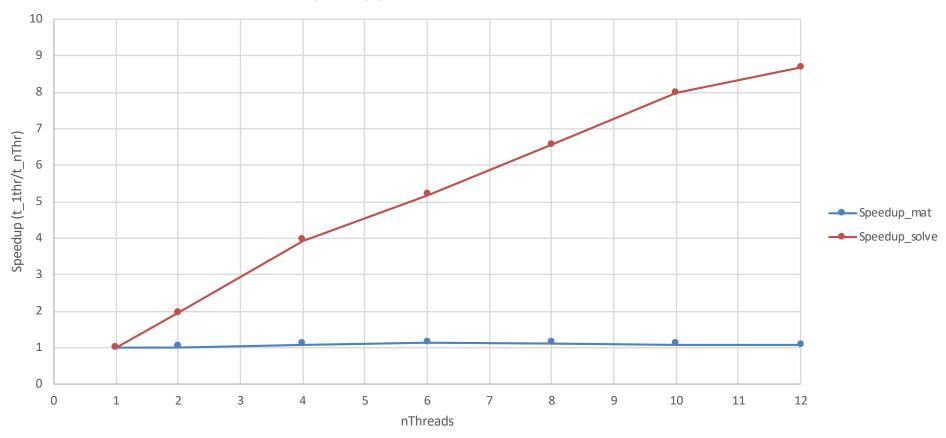


- Run panel3d code with no parallelization
- Measured on HoRUS, 1 node
- Varied N
- Varied maxNumCompThreads
- Recorded compute time with tic and toc
- Read CPU and RAM use from top





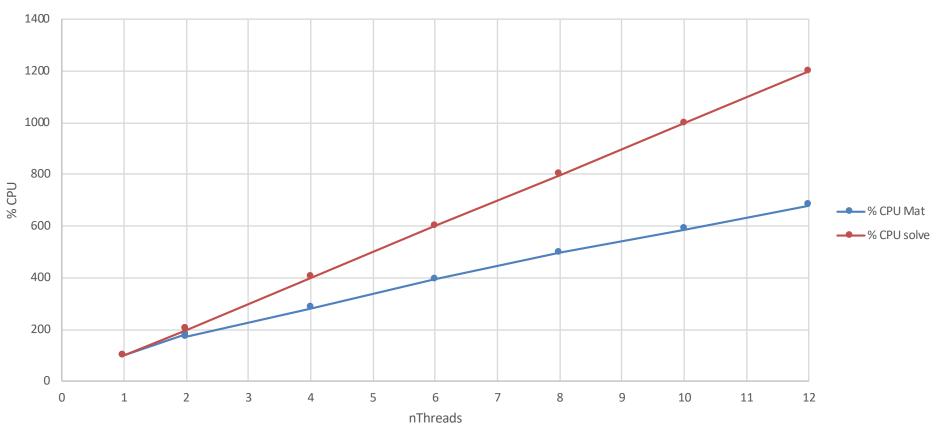
Speedup panel3d with MATLAB. N=10000







CPU load running panel3d with MATLAB, N=10000







- Speed:
 - -Building up the matrices: barely scales with threads
 - Best performance at 6 cores (number of cores on 1 socket)
 - -Solve LES: scales almost perfectly (ScaLAPACK shipped with MATLAB)
- Memory
 - -LES buildup: memory overhead present but reasonable
 - CLI MATLAB has about 650 MB base RAM usage
 - Total measured: 2.4 GB RAM (theoretical 1.6 GB)
 - -LES solve: 3.2 GB RAM
 - Algorithm?





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Parallel performance

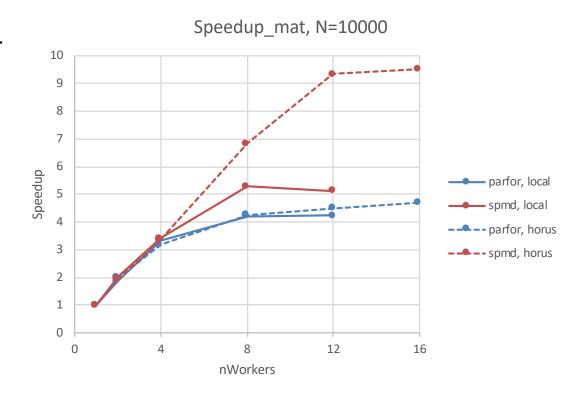
- Divide work up to workers
 - –Building LES
 - –Solving LES
- Try different strategies
 - -parfor
 - -spmd
 - Local vs. Horus custer profile
- N = 10000, Number of workers 1, 2, 4, 8, 12, 16
- Determine speedup





Speedup of matrix building operations

- Speedup has plateau
 - -Local profile: much earlier
- Most likely some unparallelizable operations
 - –Does not happen with N = 20000
- spmd-based operation performs better
 - -Unclear cause

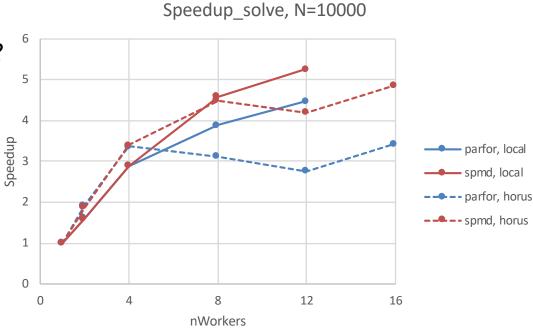






Speedup of matrix building operations

- Starts to plateau with local profile
 - -Unclear cause
 - -Communication overhead of alg.?
- Considerably lower performance with parfor and Horus profile
 - -Strange: this is always an spmd,
 not a parfor
- Reason:
 - -Automatic array distribution in parfor
 - -Needs to be redistributed







Parallel Performance

• Both parfor and spmd accelerate calculation

All less than n times serial

• spmd slightly faster

- Larger matrices possible with spmd
 - -Likely due to distributed matrices





mpiprofile

• At this point, just a shoutout to mpiprofile

• I have not seen such a comfortable tool for parallel performance measurement in any language

Demo 10





spmd Lessons Learned

- Always keep track of whether a variable is distributed
- Assigning will overwrite
 - -Assigning like a(:) = b will give error when b is distributed
- Old Matlab rule about matrix operations rather than loops still holds
 - -Setting and accessing entries: prohibitively expensive
 - -Better: copy to local variable
- But: basically every Matlab function works with distributed arrays (!)
 - -Unsure which matrix distribution is best





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Exercise 4

- Objectives
 - You can use the different profiling methods
 - You can interpret serial and parallel profiler outputs
- Tasks
 - Add the commented profiling commands in the spmd example (Appendix B) one by one, run the code and examine the output
 - tic and toc
 - profile
 - mpiprofile

Note the following slide (before you begin the exercise)





Exercise 4

- Notes
 - -You can use the return command inside a script to exit it early
- If bored, get creative
 - –Add different functions and operation to the statements
 - -Start profiling your own MATLAB code
 - Implement and play around with the built-in function example (Appendix C)

— ...





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What to do about this

- Matrix calculation was tough to optimize
 - –Limits of MATLAB reached?

• Idea: delegate matrix calculation function to external Fortran program

MATLAB has interface for this

• MATLAB extension function: Mex





Mex interface

- MATLAB has loads of interfaces
 - -Primarily to C and C++
 - Also Python
 - -Also Fortran
 - -Both ways (run MATLAB from another language)
- Slightly confusing naming principle
 - -Matrix vs mex interface?
 - -Call libraries vs. call executables
- We will only look at Fortran mex interface

https://www.mathworks.com/help/matlab/external-language-interfaces.html





Mex interface

- To investigate:
 - -Fortran or C (or C++) interface more comfortable?
 - -How much programming knowledge needed?
 - -How well integrated is mex?
 - How much better is performance
- What is mex exactly?
 - –Interface to system C or Fortran compiler (Linux: gcc)
 - –Stand-alone executable (outside MATLAB)
 - -Mex interface: list of C and Fortran functions to put in own code





Interfacing Fortran code with MATLAB

- Strategy:
 - -Matrix calculation slow and badly parallelized
 - -panel3d Fortran version at least twice as fast as fastest MATLAB version
 - → Use calcMats() function from Fortran
- What needs to be given to Fortran
 - -Input: InflData object
 - –Output: B and C matrices
- Idea: implement this interface in Fortran (code exists)
 - -Fortran because matrix syntax similar to MATLAB
 - -Future: Also C and/or C++





Interfacing Fortran code: approach

- Mex principle:
 - -Single Fortran routine called mexFunction() acts as entry/exit point
 - Use as wrapper for existing own code
 - Decode arguments

- What needs to be given to Fortran
 - -Input: InflData object
 - -Output: B and C matrices





Interfacing Fortran code: own code

- Fortran code very similar to MATLAB
- Did not take long to convert
- Parallelization in this case similar to parfor logic
- Row-major vs. columnmajor more important





Hello World mexFunction breakdown

- Single include
- Pointer to left-hand (output) and right-hand-side (input) args
 - –Number of args
 - -Requires checks for everything
- Pointer data type
- mex-specific library function

```
#include "fintrf.h"
subroutine mexFunction(nlhs. plhs, nrhs, prhs)

implicit none

mwPointer :: plhs(*), prhs(*)
integer :: nlhs, nrhs

call mexPrintf( "Hello MATLAB world, this is Fortran!\n" )
end subroutine mexFunction
```





Decoding and re-encoding arguments

- This function gets one array from args
- Requires 4 steps
- Requires copying array
 - Have not found better way
- Objects: repeat this for every member variable

But: works, not complex if you understood it once





Simplifying mex interface

- MATLAB File Exchange: Fortran 95 interface "with extras"
 - https://www.mathworks.com/matlabcentral/fileexchange/25934-fortran-95-interface-to-matlab-api-with-extras
 - -Community effort, not officially supported
 - Have not tried yet
- Also: pointer interface is C logic
 - -Clearly, C support for mex came first
 - More comfortable?
 - Problem: matrix handling in C less comfortable





MATLAB C++ Interfaces

- Multiple C++ APIs, confusing naming
 - -C++ mex API: define C++ functions, call them from MATLAB
 - -C++ <u>Data</u> API: use MATLAB data structures in C++ code (arrays)
 - -C++ **Engine** API: talk to MATLAB from C++ (call functions)
 - -<u>C</u> API: separate, not covered
- Performance not yet tested, therefore only how comfortable is it to use?





MATLAB C++ Interfaces

- Advantages:
 - -C++ features very clean and well designed (understood it quickly)
 - Much more comfortable for C++ programmers
 - Object-oriented approach, including exceptions
- Disadvantages:
 - –C++ more strict about datatypes (Fortran too by the way)
 - Unclear how much overhead
 - Resulting arrays are not C++-native, limited by implementation
 - No functions like sin, cos, sqrt
 - Everything needs to be a loop





MATLAB C++ Interfaces

Headers come with MATLAB

Practical namespaces

```
#include "mex.hpp"
#include "mexAdapter.hpp"
using matlab::engine::MATLABEngine;
using matlab::data::Array;
typedef matlab::data::TypedArray<double> DblArray;
typedef matlab::data::TypedArray<bool> BoolArray;
typedef matlab::data::TypedArray<int64_t> Int32Array;
class MexFunction : public matlab::mex::Function {
public:
    void operator()(matlab::mex::ArgumentList outputs, matlab::mex::ArgumentList inputs)
        std::cout << "Hello MATLAB world, this is C++!" << std::endl;</pre>
        const Array& inflIn = inputs[0];
        const DblArray& xTarget = inputs[1];
        const Engine engine = getEngine();
        InflData infl( engine, inflIn );
        infl.calc(xTarget);
```





Code comparison

MATLAB

```
function pVec = calcPVectors(centers,xTarget,pVec)
    pVec(:,:) = xTarget - centers;
end
```

Fortran





Code comparison

MATLAB

```
function pVec = calcPVectors(centers,xTarget,pVec)
    pVec(:,:) = xTarget - centers;
end
```

C++

```
static void calcPVectors( const DblArray& centers, const DblArray& xTarget, DblArray& pVec)
{
    for (size_t i = 0; i < pVec.getDimensions()[0]; i++)
    {
        for (size_t j = 0; j < pVec.getDimensions()[1]; j++ )
        {
            pVec[i][j] = xTarget[0][j] - centers[i][j];
            std::cout << pVec[i][j] << " ";
        }
        std::cout << std::endl;
}</pre>
```





Calling mex

- Simple command line tool
 - -Already available on cluster
 - -Even cmake knows MATLAB and mex (!)
- One call:

```
mex -I../Release horus ../src/mex/calcMatsMex.F90
```

- Include directory with own code
 - Needs to be compiled separately
- Easy setup compared to usual C or Fortran compilers
 - -Complexity is only in the way interface works





Summary of mex experiments

- MATLAB code can potentially benefit greatly from external code
 - -Several times faster
 - -Complete arsenal of optimizations (vector registers etc.)
 - -May parallelize with OpenMP and MPI
- Fortran interface is extremely awkward
 - -Clearly a C relic
- Getting mex to run is however very simple
 - Already installed
 - -Minimal includes/library links etc.





Recommendations for mex use

- Minimize interface interaction
 - –Few input/output arguments
 - –Simple types (arrays)

- Choose interface wisely
 - -Which language do you know better?

- Good candidates for delegation to other language
 - -Function that takes some arrays, has big loop(s), outputs some other arrays





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Summary

- Wide variety of parallel options
 - Comfortable profiling and debugging (mostly)
- Built-in functions often already optimized
 - -Two orders of magnitude when written in another language
- · Built-in multithreading
 - –Helps a lot
 - "Free" (no additional coding)
- parfor
 - Easy to implement
 - -Not transparent





Summary

- spmd
 - Lots and lots of possibilities
 - -Complex
 - Lots of potential for errors
- Scaling good for built-in functions, bad for own functions
- mex
 - -Initially tricky
 - -Ultimately not hard
 - -Potentially huge benefit
 - -Fortran: code very similar





Recommendations

- Best approach for writing your code:
 - 1. Use MATLAB for code until it works (rapid prototyping)
 - Focuses on mathematical operations
 - Comfortable debugging
 - Built-in visualization
 - 2. Once it works, analyze performance
 - 3. Identify functions where most time is spent
 - Simple in programming terms (likely complex in mathematical terms)
 - Big long loops inside
 - 4. Delegate these to mex function
 - Feasible(!) after initial complexity
 - Fortran: similar to MATLAB syntax, C closer to interface logic





Thank you for your attention

Questions?





Feedback round

- What were your expectations, and where they fulfilled?
- What was your favorite part about the course?
- What did you dislike or what do you feel can be improved?
- How did you learn about this course?
- What other topics would you like to see in future ZIMT courses?





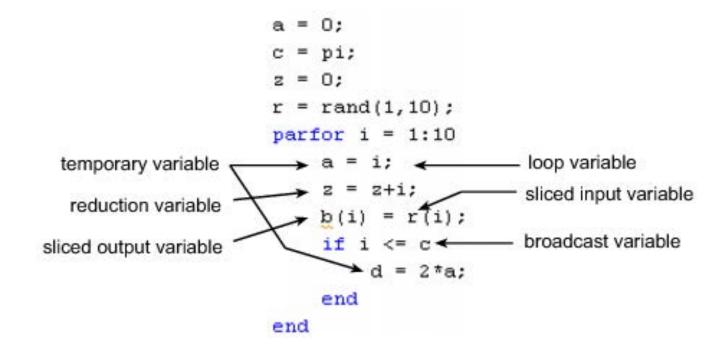
Appendices: code examples

- Appendix A: parfor example
- Appendix B: SPMD test example
- Appendix C: built-in functions performance example





Appendix A: parfor example







Appendix B: spmd example

```
n = 10000;
 % maxNumCompThreads(2);
 maxNumCompThreads
 As = rand(n);
 rs = rand(n,1);
 % profile on
 % tic
 xs = As \setminus rs;
 xs = vecnorm(As, 2, 1);
 % toc
 % profile viewer
□ spmd
       codist1 = codistributor2dbc(codistributor2dbc.defaultLabGrid, ...
                                 codistributor2dbc.defaultBlockSize, ...
     codist1 = codistributor1d(1);
     Ap = rand(n,codist1);
     rp = rand(n,1,codist1);
      mpiprofile on
      tic
     xp = Ap \setminus rp;
     x = vecnorm(A, 2, 2);
      mpiprofile off
      mpiprofile viewer;
```





Appendix C: built-in functions performance example

```
clear;
n = 20000;
A = rand(n);
b = rand(n,1);
c = zeros(1,n);
% Without reallocating c
c(:) = A * b;
% With possible reallocation of c
c = A * b;
% With newly allocated result
c2 = A * b;
Manual matrix multiplication
tic
for i = 1:n
   for j = 1:n
       c(i) = c(i) + A(i,j) * b(j);
% Manual matrix multiplication with reallocation in each step
for i = 1:n
    c3(i) = 0.0;
    for j = 1:n
        c3(i) = c3(i) + A(i,j) * b(j);
```